www.kvi.nl/~gohari

Subatomic Physics

Final exam

Date: Monday, Feb. 3, 2003

This exam has a total of 100 points

Problem #1. *(10 points)*

a) Given that the form factor of a nucleus can be expressed as the Fourier transform of the charge-density distribution

$$F(q) = \int d^3R\ \rho(\vec{R})\ e^{-i\vec{q}\cdot\vec{R}}$$

and assuming a spherical shape for the charge-density distribution, perform an expansion around small momentum transfers and explain the first three terms that you obtain in terms of physical quantities (moments). *(6 points)* 6

b) Calculate the relation between the root-mean-square radius R_{rms} and the radius R of a homogenouosly-charged sphere with charge Ze. *(4 points)*

Problem #2. *(15 points)*

a) Write down the Bethe-Weizsäcker empirical mass formula (you don't need to know the constants. Here, they are given for your information: $a_V = 15.85$ MeV, $a_S = 18.34$ MeV, $a_a = 23.21$ MeV, $a_c = 0.71$ MeV and $a_p = 12$ MeV). *(4 points)*

b) Why is the form parabolic in Z for large Z? Elaborate by making an estimation leading to this term. *(4 points)* 11

c) Calculate the relation between the mass number A and the charge number Z for the most stable nuclei assuming large A (line of stability). *(4 points)*

d) For a constant *odd* A, draw a picture of the binding energy as a function of Z and show how the stable isotope is reached and through which decay paths (specify the decay type as well). *(3 points)*

0 **Problem #3.** *(5 points)*

The nuclear spin-orbit coupling is proportional to the expectation value of $\vec{l}\cdot\vec{s}$. Express $\vec{l}\cdot\vec{s}$ in terms of j, l and s. Show that the energy separation of a nuclear spin-orbit doublet is proportional to $\frac{1}{2}(2l+1)$ (Note that $s = 1/2$ for this). 5

Problem #4. *(8 points)*
Use the accompanying scheme for the following cases:

a) The low-lying levels of $^{39}_{20}$Ca have spin-parity values, starting from the ground state, of $\frac{3}{2}^+$, $\frac{1}{2}^+$, $\frac{7}{2}^-$, and $\frac{3}{2}^-$. What are the reasons for agreements or disagreements with the single-particle shell model? *(4 points)* 8

b) The low-lying levels of $^{207}_{82}$Pb have spin-parity values, starting from the ground state, of $\frac{1}{2}^-$, $\frac{5}{2}^-$, $\frac{3}{2}^-$, $\frac{13}{2}^+$ and $\frac{7}{2}^-$. What are the reasons for agreements or disagreements with the single-particle shell model? *(4 points)*

$\frac{\hbar^2}{2I}\left(J(J+1)-K^2\right)$

Problem #5. *(9 points)*
The nucleus ^{176}Hf has ground and excited rotational state energy levels of: 0^+ (0.0 MeV), 2^+ (0.088 MeV), 4^+ (0.290) MeV. Note: these are the energies of the levels.

a) Deduce the moment of inertia for this nucleus (in units of MeV^{-1}). Compare this value with the moment of inertia assuming the nucleus to be rigid rotor. The moment of inertia of rigid rotor with mass M and radius R is $\frac{2}{5}MR^2$. *(4 points)* 1.2 fm 9

b) What would you predict for the energy of the 6^+ state? *(3 points)*

c) When the spin of the nucleus is very large the moment of inertia increases as a function of spin. Explain the effect. *(2 points)* Centrifugal Stretching.
even higher J → backbending, breaking of pairs.

Problem #6. *(5 points)*

a) What is the energy dependence of the beta decay spectrum in a Kurie or Fermi plot for massless neutrinos. The Kurie plot is $\left(\frac{d\omega}{p_e^2 dp_e}\right)^{1/2}$ as a function of the electron energy. *(3 points)*

b) What happens in this plot when the neutrino is massive? Show this on a plot (Do NOT derive the formulas). *(2 points)* 2

Problem #7. *(8 points)*
The basic form of the Yukawa potential can be understood by considering the exchange of a spin-0 boson with mass m, obeying the static ~~Kelin~~ Klein-Gordon equation:

$$(\nabla^2 - m^2c^2/\hbar^2)\Phi(r) = 0$$

a) Show that $\Phi(r) = V_0 \cdot \exp(-r/R)/(r/R)$ is a good solution (you can ignore the angular part of the equation and replace ∇^2 by $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r})$). *(4 points)*

b) What is R? Give a physical interpretation for it and obtain it from the uncertainty principal. *(4 points)*

Problem #8. *(10 points)*

a) Show that a measurement of any pseudoscalar observables must be an evidence of parity violation (initial and final states have the same parity). *(5 points)*

b) If parity is conserved. prove that the expectation value of any operator that changes sign under parity operation is zero unless the initial and final states have opposite parities. *(5 points)*

Problem #9. *(10 points)*

a) Write down the Gell-Mann Nishijima relationship relating the charge to isospin, Baryon number and strangeness. *(4 points)*

b) The Σ baryon exists in 3 charge states. Show that the strangeness quantum number of this particle is $S = -1$. *(3 points)*

c) The Ξ baryon exists in 2 charge states (-1 and 0). Show that the strangeness quantum number of this particle is $S = -2$. *(3 points)*

Problem #10. *(14 points)*

Positronium is an atom which consists of an electron and a positron (e^+e^-).

a) Using the generalized Pauli principle, show the symmetry property of this atom when operated by the charge-conjugation operator, C. *(6 points)*

b) Using the above property, write down what the spin and orbital angular momentum have to satisfy in order that this atom decay into

   1) 2 photons; *(3 points)*
   2) 3 photons. *(3 points)*

c) Specify the spin states for both cases when $l = 0$. *(2 points)*

Problem #11. *(6 points)*

Which of the following reactions are allowed and which are forbidden? If allowed, explain which interaction and if not, why not? *(1 point for each reaction)*

1. $p\pi^- \to pK^-$
2. $\nu_e p \to e^-\Sigma^+K^+$
3. $p\bar{p} \to \pi^+\pi^-\pi^0\pi^+\pi^-$
4. $\Xi^0 \to \Sigma^0\gamma$
5. $\mu^- + p \to \Lambda^0 + \nu_\mu$
6. $K^+ \to \pi^0 e^+\nu_e$